Hierarchical Stick-breaking Feature Paintbox

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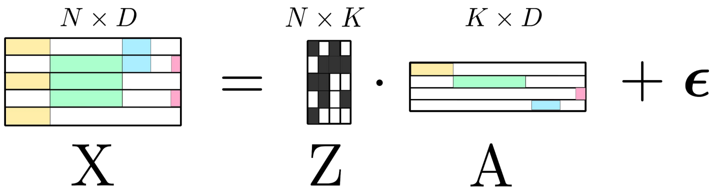
We present a ***latent feature model*** with a ***flexible nonparametric prior*** allowing for ***arbitrary correlations*** amongst features and tractable inference.

Feature Model



$$\boldsymbol{\nu} \sim \mathrm{HSBP}(\alpha, p) \qquad \boldsymbol{z}_n \sim \mathrm{Mult}\left(1, \{\pi_{\boldsymbol{\epsilon}}\}_{\boldsymbol{\epsilon} \in \mathcal{S}_K}\right)$$

$$\mathbf{A} \sim \mathcal{N}(0, \sigma_0^2 \mathbf{I}) \qquad \mathbf{X}|\mathbf{Z}, \mathbf{A} \sim \mathcal{N}(\mathbf{ZA}, \sigma_x^2 \mathbf{I}),$$

$$p(\boldsymbol{z}_n) = \prod_{k=1}^{K} p\left(z_{nk} | \boldsymbol{z}_{n,1:(k-1)}\right) = \prod_{k=1}^{K} \mathrm{Bernoulli}(\nu_{\epsilon_k})$$

Construction of the Prior

- $\pi_{\emptyset} = 1$, $\nu_{\emptyset} \sim \mathrm{Beta}(\frac{\alpha}{K^p}, 1)$
- $\forall\, k = 1, \cdots, K$, and $j = 1, \cdots, 2^{k-1}$, draw $\nu_{\epsilon_j} \sim \mathrm{Beta}(\frac{\alpha}{K^p}, 1)$, such that:

$$\begin{aligned}
\pi_1 &= \nu_{\emptyset} \\
\pi_0 &= (1 - \nu_{\emptyset}) \\
\pi_{01} &= (1 - \nu_{\emptyset})\nu_1 \\
\pi_{111} &= \nu_{\emptyset}\nu_1\nu_{11} \\
\pi_{010} &= (1 - \nu_{\emptyset})\nu_1(1 - \nu_{01}) \\
&\ldots
\end{aligned}$$

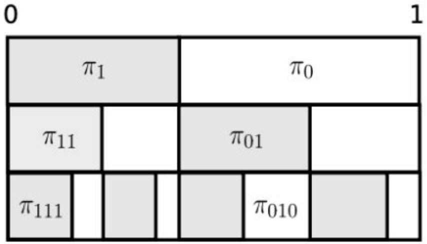


Figure 1: Example of canonical paintbox.

Inference

- collapsed Gibbs sampler

$$p\left(z_{nk}|\mathbf{Z}_{-(nk)}\right) \propto \prod_{\epsilon\in\mathcal{S}_n} \frac{\left(\frac{\alpha}{K^p}+\phi_{\epsilon 1}^{-n}\right)^{z_{nk}}\left(1+\phi_{\epsilon 0}^{-n}\right)^{(1-z_{nk})}}{\left(\frac{\alpha}{K^p}+1+\phi_{\epsilon}^{-n}\right)}$$

Model Properties

1. Non-degeneracy

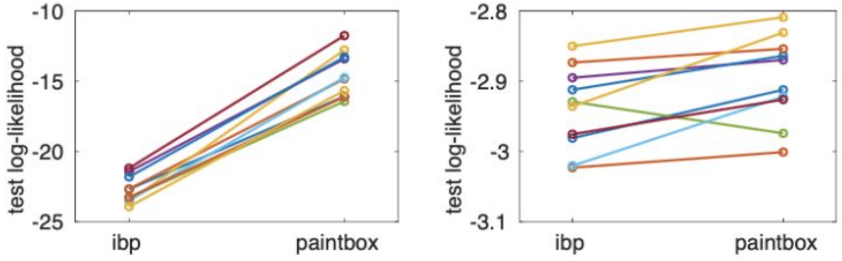
$$\lim_{K\to\infty} \mathbb{E}\left[\pi_K\right]=0$$

2. Exchangeability

$$p(\boldsymbol{z}_2,\boldsymbol{z}_3|\boldsymbol{z}_1) \stackrel{d}{=} p(\boldsymbol{z}_3,\boldsymbol{z}_2|\boldsymbol{z}_1)$$

Results

1. Improved Performance on Held-out Data



(a) correlated toy images (b) breast cancer

2. Improved Recovery of Latent Features